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06CS42

Fourth Semester B.E. Degree Examination, December 2010

Graph Theory and Combinatories

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Define: i) Connected graph ii) Spanning subgraph, and iii) Complement of a graph. Give one example for each. **(06 Marks)**

b. Explain, with an example, graph isomorphism. Show that in a graph G, the number of odd degree vertices is even. **(07 Marks)**

c. Write a note on "Konigsberg-bridge problem". **(07 Marks)**

2 a. Define complete bipartite graph. Prove that Kuratowski's second graph $K_{3,3}$, is nonplanar. **(06 Marks)**

b. Show that in any connected planar graph with 'n' vertices, 'e' edges and 'f' faces, e-n+2=f (Euler's formula). **(07 Marks)**

c. Define chromatic number and chromatic polynomial. Find the chromatic polynomial for the graph given below: **(07 Marks)**

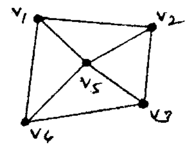

Fig. Q2 (c)

3 a. Define : i) Tree ii) Binary rooted tree, and iii) Prefix code. Give one example for each. **(06 Marks)**

b. Prove that a tree with 'n' vertices has (n-1) edges. **(07 Marks)**

c. Find all the spanning trees of the graph given below: **(07 Marks)**

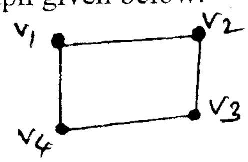

Fig. Q3 (c)

4 a. Define : i) Matching ii) Complete matching and iii) Edge-connectivity (with example). **(06 Marks)**

b. Find a minimal spanning tree using prims algorithm for the weighted graph given below: **(07 Marks)**

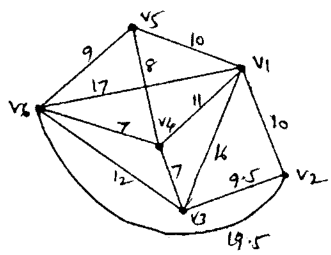

Fig. Q4 (b)

4 c. Find the maximum flow possible between the vertices A and D for the following graph: **(07 Marks)**

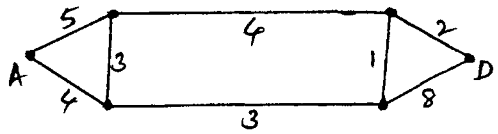

Fig. Q4 (c)

PART – B

5 a. In how many ways can one distribute 10 identical white marbles among six distinct containers? **(06 Marks)**

b. i) How many 9 letter words can be formed using the letters of the word "Difficult"?

ii) A certain question paper contains two parts A and B each having 4 questions. How many different ways a student can answer 5 questions by selecting at least two questions from each part? **(07 Marks)**

c. Let a triangle ABC be equilateral, with AB = 1. Show that if we select 10 points in the interior of this triangle, there must be at least two points, whose distance apart is less than $\frac{1}{3}$. **(07 Marks)**

6 a. In how many ways can the integers 1, 2, 3, ...10 be arranged in a line, so that, no even integer is in its natural place? **(06 Marks)**

b. In how many ways can one arrange the letters in CORRESPONDENTS so that:

i) There are exactly two pairs of consecutive identical letters.

ii) There are at least three pairs of consecutive identical letters. **(07 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish to have the apple, the boy B_3 does not want the banana or mango and B_4 returns the orange. In how many ways the distribution can be made so that no boy is displeased? **(07 Marks)**

7 a. Let $f(x) = (1+x+x^2)(1+x)^n$, where $n \in z^+$. Find the coefficient of the following in the expansion of f(x):

i) x^7 ii) x^8 iii) x^r, $0 \le r \le (n+2)$, $r \in z$. **(06 Marks)**

b. In how many ways can 12 oranges be distributed among three children A, B, C so that A gets at least four, B and C get at least two but C gets no more than five? **(07 Marks)**

c. Find the exponential generating function for the number of ways to arrange 'n' letters, $n \ge 0$, selected from each of the following words:

i) HAWAII ii) MISSISSIPPI iii) ISOMORPHISM **(07 Marks)**

8 a. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(06 Marks)**

b. Solve the recurrence relation,

$a_n - 6a_{n-1} + 9a_{n-2} = 0$, $n \ge 2$, given $a_0 = 5$, $a_1 = 12$ **(07 Marks)**

c. Find the generating function for the recurrence relation,

$a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \ge 0$, with $a_0 = 3$, $a_1 = 7$. Hence solve it. **(07 Marks)**

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